

INSTRUCTOR'S SOLUTIONS MANUAL

PROBABILITY AND STATISTICAL INFERENCE

NINTH EDITION

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PEARSON

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Preface

This solutions manual provides answers for the even-numbered exercises in *Probability and Statistical Inference*, 9th edition, by Robert V. Hogg, Elliot A. Tanis, and Dale L. Zimmerman. Complete solutions are given for most of these exercises. You, the instructor, may decide how many of these solutions and answers you want to make available to your students. Note that the answers for the odd-numbered exercises are given in the textbook.

All of the figures in this manual were generated using *Maple*, a computer algebra system. Most of the figures were generated and many of the solutions, especially those involving data, were solved using procedures that were written by Zaven Karian from Denison University. We thank him for providing these. These procedures are available free of charge for your use. They are available for download at <http://www.math.hope.edu/tanis/>. Short descriptions of these procedures are provided on the “Maple Card.” Complete descriptions of these procedures are given in *Probability and Statistics: Explorations with MAPLE*, second edition, 1999, written by Zaven Karian and Elliot Tanis, published by Prentice Hall (ISBN 0-13-021536-8). You can download a copy of this manual at <http://www.math.hope.edu/tanis/MapleManual.pdf>.

Our hope is that this solutions manual will be helpful to each of you in your teaching.

If you find an error or wish to make a suggestion, send these to Elliot Tanis, tanis@hope.edu, and he will post corrections on his web page, <http://www.math.hope.edu/tanis/>.

R.V.H.
E.A.T.
D.L.Z.

Chapter 1

Probability

1.1 Properties of Probability

1.1-2 Sketch a figure and fill in the probabilities of each of the disjoint sets.

Let $A = \{\text{insure more than one car}\}$, $P(A) = 0.85$.

Let $B = \{\text{insure a sports car}\}$, $P(B) = 0.23$.

Let $C = \{\text{insure exactly one car}\}$, $P(C) = 0.15$.

It is also given that $P(A \cap B) = 0.17$. Since $A \cap C = \phi$, $P(A \cap C) = 0$. It follows that $P(A \cap B \cap C') = 0.17$. Thus $P(A' \cap B \cap C') = 0.06$ and $P(A' \cap B' \cap C) = 0.09$.

1.1-4 (a) $S = \{\text{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT}\}$;

(b) (i) $5/16$, (ii) 0 , (iii) $11/16$, (iv) $4/16$, (v) $4/16$, (vi) $9/16$, (vii) $4/16$.

1.1-6 (a) $P(A \cup B) = 0.4 + 0.5 - 0.3 = 0.6$;

$$\begin{aligned} \text{(b)} \quad A &= (A \cap B') \cup (A \cap B) \\ P(A) &= P(A \cap B') + P(A \cap B) \\ 0.4 &= P(A \cap B') + 0.3 \\ P(A \cap B') &= 0.1; \end{aligned}$$

$$\text{(c)} \quad P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.3 = 0.7.$$

1.1-8 Let $A = \{\text{lab work done}\}$, $B = \{\text{referral to a specialist}\}$,

$$P(A) = 0.41, P(B) = 0.53, P[(A \cup B)'] = 0.21.$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.79 = 0.41 + 0.53 - P(A \cap B)$$

$$P(A \cap B) = 0.41 + 0.53 - 0.79 = 0.15.$$

$$\text{1.1-10} \quad A \cup B \cup C = A \cup (B \cup C)$$

$$P(A \cup B \cup C) = P(A) + P(B \cup C) - P[A \cap (B \cup C)]$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)]$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C)$$

$$+ P(A \cap B \cap C).$$

1.1-12 (a) $1/3$; (b) $2/3$; (c) 0 ; (d) $1/2$.

$$1.1-14 \quad P(A) = \frac{2[r - r(\sqrt{3}/2)]}{2r} = 1 - \frac{\sqrt{3}}{2}.$$

1.1-16 Note that the respective probabilities are p_0 , $p_1 = p_0/4$, $p_2 = p_0/4^2, \dots$.

$$\begin{aligned} \sum_{k=0}^{\infty} \frac{p_0}{4^k} &= 1 \\ \frac{p_0}{1 - 1/4} &= 1 \\ p_0 &= \frac{3}{4} \\ 1 - p_0 - p_1 &= 1 - \frac{15}{16} = \frac{1}{16}. \end{aligned}$$

1.2 Methods of Enumeration

$$1.2-2 \quad \text{(a)} \quad (4)(5)(2) = 40; \quad \text{(b)} \quad (2)(2)(2) = 8.$$

$$1.2-4 \quad \text{(a)} \quad 4 \binom{6}{3} = 80;$$

$$\text{(b)} \quad 4(2^6) = 256;$$

$$\text{(c)} \quad \frac{(4 - 1 + 3)!}{(4 - 1)!3!} = 20.$$

1.2-6 $S = \{ \text{DDD, DDFD, DFDD, FDDD, DFFD, DFDF, FDDF, DFFD, FDFD, FDFD, FFFF, FDFD, DFFF, FDFD, DFFF, FDFD, DFFF, FDFD} \}$ so there are 20 possibilities.

$$1.2-8 \quad 3 \cdot 3 \cdot 2^{12} = 36,864.$$

$$\begin{aligned} 1.2-10 \quad \binom{n-1}{r} + \binom{n-1}{r-1} &= \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!} \\ &= \frac{(n-r)(n-1)! + r(n-1)!}{r!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}. \end{aligned}$$

$$1.2-12 \quad 0 = (1-1)^n = \sum_{r=0}^n \binom{n}{r} (-1)^r (1)^{n-r} = \sum_{r=0}^n (-1)^r \binom{n}{r}.$$

$$2^n = (1+1)^n = \sum_{r=0}^n \binom{n}{r} (1)^r (1)^{n-r} = \sum_{r=0}^n \binom{n}{r}.$$

$$1.2-14 \quad \binom{10-1+36}{36} = \frac{45!}{36!9!} = 886,163,135.$$

$$1.2-16 \quad \text{(a)} \quad \frac{\binom{19}{3} \binom{52-19}{6}}{\binom{52}{9}} = \frac{102,486}{351,325} = 0.2917;$$

$$\text{(b)} \quad \frac{\binom{19}{3} \binom{10}{2} \binom{7}{1} \binom{3}{0} \binom{5}{1} \binom{2}{0} \binom{6}{2}}{\binom{52}{9}} = \frac{7,695}{1,236,664} = 0.00622.$$

1.3 Conditional Probability

1.3-2 (a) $\frac{1041}{1456}$;

(b) $\frac{392}{633}$;

(c) $\frac{649}{823}$.

(d) The proportion of women who favor a gun law is greater than the proportion of men who favor a gun law.

1.3-4 (a) $P(\text{HH}) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$;

(b) $P(\text{HC}) = \frac{13}{52} \cdot \frac{13}{51} = \frac{13}{204}$;

(c) $P(\text{Non-Ace Heart, Ace}) + P(\text{Ace of Hearts, Non-Heart Ace})$
 $= \frac{12}{52} \cdot \frac{4}{51} + \frac{1}{52} \cdot \frac{3}{51} = \frac{51}{52 \cdot 51} = \frac{1}{52}$.

1.3-6 Let $H = \{\text{died from heart disease}\}$; $P = \{\text{at least one parent had heart disease}\}$.

$$P(H | P) = \frac{N(H \cap P)}{N(P)} = \frac{110}{648}.$$

1.3-8 (a) $\frac{3}{20} \cdot \frac{2}{19} \cdot \frac{1}{18} = \frac{1}{1140}$;

(b) $\frac{\binom{3}{2} \binom{17}{1}}{\binom{20}{3}} \cdot \frac{1}{17} = \frac{1}{380}$;

(c) $\sum_{k=1}^9 \frac{\binom{3}{2} \binom{17}{2k-2}}{\binom{20}{2k}} \cdot \frac{1}{20-2k} = \frac{35}{76} = 0.4605$.

(d) Draw second. The probability of winning is $1 - 0.4605 = 0.5395$.

1.3-10 (a) $P(A) = \frac{52}{52} \cdot \frac{51}{52} \cdot \frac{50}{52} \cdot \frac{49}{52} \cdot \frac{48}{52} \cdot \frac{47}{52} = \frac{8,808,975}{11,881,376} = 0.74141$;

(b) $P(A') = 1 - P(A) = 0.25859$.

1.3-12 (a) It doesn't matter because $P(B_1) = \frac{1}{18}$, $P(B_5) = \frac{1}{18}$, $P(B_{18}) = \frac{1}{18}$;

(b) $P(B) = \frac{2}{18} = \frac{1}{9}$ on each draw.

1.3-14 (a) $P(A_1) = 30/100$;

(b) $P(A_3 \cap B_2) = 9/100$;

(c) $P(A_2 \cup B_3) = 41/100 + 28/100 - 9/100 = 60/100$;

$$(d) P(A_1 | B_2) = 11/41;$$

$$(e) P(B_1 | A_3) = 13/29.$$

$$1.3-16 \quad \frac{3}{5} \cdot \frac{5}{8} + \frac{2}{5} \cdot \frac{4}{8} = \frac{23}{40}.$$

1.4 Independent Events

$$1.4-2 \quad (a) \quad \begin{aligned} P(A \cap B) &= P(A)P(B) = (0.3)(0.6) = 0.18; \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.6 - 0.18 \\ &= 0.72. \end{aligned}$$

$$(b) \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{0.6} = 0.$$

$$1.4-4 \quad \text{Proof of (b):} \quad \begin{aligned} P(A' \cap B) &= P(B)P(A'|B) \\ &= P(B)[1 - P(A|B)] \\ &= P(B)[1 - P(A)] \\ &= P(B)P(A'). \end{aligned}$$

$$\text{Proof of (c):} \quad \begin{aligned} P(A' \cap B') &= P[(A \cup B)'] \\ &= 1 - P(A \cup B) \\ &= 1 - P(A) - P(B) + P(A \cap B) \\ &= 1 - P(A) - P(B) + P(A)P(B) \\ &= [1 - P(A)][1 - P(B)] \\ &= P(A')P(B'). \end{aligned}$$

$$1.4-6 \quad \begin{aligned} P[A \cap (B \cap C)] &= P[A \cap B \cap C] \\ &= P(A)P(B)P(C) \\ &= P(A)P(B \cap C). \end{aligned}$$

$$\begin{aligned} P[A \cap (B \cup C)] &= P[(A \cap B) \cup (A \cap C)] \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \\ &= P(A)[P(B) + P(C) - P(B \cap C)] \\ &= P(A)P(B \cup C). \end{aligned}$$

$$\begin{aligned} P[A' \cap (B \cap C')] &= P(A' \cap C' \cap B) \\ &= P(B)[P(A' \cap C') | B] \\ &= P(B)[1 - P(A \cup C | B)] \\ &= P(B)[1 - P(A \cup C)] \\ &= P(B)P[(A \cup C)'] \\ &= P(B)P(A' \cap C') \\ &= P(B)P(A')P(C') \\ &= P(A')P(B)P(C') \\ &= P(A')P(B \cap C'). \end{aligned}$$

$$\begin{aligned} P[A' \cap B' \cap C'] &= P[(A \cup B \cup C)'] \\ &= 1 - P(A \cup B \cup C) \\ &= 1 - P(A) - P(B) - P(C) + P(A)P(B) + P(A)P(C) + \\ &\quad P(B)P(C) - P(A)P(B)P(C) \\ &= [1 - P(A)][1 - P(B)][1 - P(C)] \\ &= P(A')P(B')P(C'). \end{aligned}$$

$$1.4-8 \quad \frac{1}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} + \frac{1}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} + \frac{5}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} = \frac{2}{9}.$$